able in both cases.‡ It is interesting to note that both the laminar theory and experiment show that at a particular Mach and Reynolds number the proper jet parameter for nondimensionalization is $(P_{0j}/P_\infty) \times (d_j/L)$. The turbulent theory indicates that P_{0j}/P_∞ is a sufficient jet parameter. The data appears to confirm this. In both cases the theory is seen to fall on the conservative side.

Conclusions

An approximate theoretical method for calculating side forces due to slot jet-boundary layer interaction has been presented. The results of sample calculations indicate that the theory predicts the correct qualitative trends and is quantitatively conservative. Finally, and most significantly, the importance of viscous mixing to the jet force interaction is brought out.

Appendix

Estimates of the bracketed term involving y_{12}/y_{34} in Eq. (6) can be made based on the experimental data of Refs. 6 and 7. Zukoski and Spaid[6] derive a relation [their Eq. (3)] for a penetration height which roughly corresponds to, and is somewhat less than, what is termed here y_{34}. Strike et al.[7] set down an empirical relation for the penetration distance of the jet shock [their empirical Eq. (1)] which roughly corresponds to y_{12}. Although these data pertain to jet injection through circular holes in plates with long downstream runs, rather than the flap-type slots under consideration here, the information is useful for estimation purposes. Table 1 contains calculations of $(y_{12}/y_{34})(P_p/P_{34})/(1 + \gamma_j M_{34}^2)$ based on expansion of the jet to P_∞, $\gamma_j = 1.4$ and use of the foregoing equations for y_{12} and y_{34}. (Turbulent flow has been assumed as this will be the more critical case.) These calculations indicate that, except for the combination of low jet pressure ratio and high Mach number (where the model applicability is questionable anyway), the assumption of $(y_{12}/y_{34})(P_p/P_{34})/(1 + \gamma_j M_{34}^2) \ll 1$ appears to be valid.

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‡ Since the submittal of this note, additional experimental data have been graciously made available by N. E. Hawk, University of Michigan. These data were taken employing side plates on a two-dimensional model and are in even better agreement with the theoretical predictions of this note than are the data shown here.

Transient Performance Prediction of a Nonadiabatic, Real-Gas Propulsion System

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Nomenclature

C_v	= heat capacity at constant volume
E	= internal energy
E_c	= $pv\dot{m}/J$, compression energy
E_t	= energy difference between inlet slug and tank gas
g	= conversion constant
H	= enthalpy
I_{ideal}	= impulse
$I_{\text{sp}_{\text{ideal}}}$	= specific impulse
J	= conversion constant
M	= mass
$\dot{m}$	= mass flow of nozzle
m	= mass increment
P	= pressure
P_e	= nozzle exit pressure
P_a	= atmospheric pressure
Q	= heat flow
S	= entropy
T	= temperature
v	= specific volume
W	= weight flow rate
α	= divergence half-angle of nozzle
ϵ	= nozzle area ratio

THE use of a gas as a propellant in a simple gas-propulsion system or as a pressurant may require the detailed analysis of the thermodynamic processes occurring within the system. The nature of most gases and the high pressures immediately preclude the use of the simplified equations representing ideal gases, and the engineer must resort to extensive tables, charts, or formidable, empirical equations of state. The analysis of the processes occurring within a gaseous system requires the determination of the state of the gas (p, v, T, H, and E), which is a difficult task for a nonideal gas that is being compressed or expanded by the addition or subtraction of gas. The difficulty is compounded when one must consider variable heat gains or losses and variable temperature mass input.

This paper presents a technique for predicting the state of any gas for which equations of state are available under the nonadiabatic transient conditions of loading (compression) or use (expansion) from a storage container. The method uses the basic differential equations of thermodynamics relating the change in internal energy, enthalpy, and entropy to the independent variables. The equations, which are put in finite-difference form, are completely general, and thus the method does not require any assumptions regarding ideality of gas. The solutions are obtained via a digital computer program, which solves the thermodynamic equations incrementally, utilizing the equations of state, the work done, the heat loss, and the mass change for inputs at each point. Since a relationship can be obtained for a nozzle for the specific impulse as a function of the state of the gas (e.g., enthalpy vs I_{sp}), then the total available impulse at any time for a system can also be determined. The validity of the method is tested by comparison of predicted temperatures and pressures to the actual values obtained during the filling and expansion of a tank with "Freon 14," a gas that is far from ideal.

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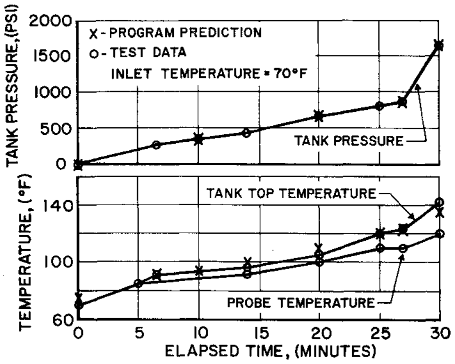

Fig. 1 Tank load test.

Analysis

The fundamental thermodynamic equations that are suitable for use in expansion and compression of gases are the following[1]:

$$dE = C_v dT + T[(\partial P/\partial T)v - P]dv \tag{1}$$

$$dH = dE + vdp + pdv = dQ + vdp \qquad dW = pdv \tag{2}$$

$$dS = (C_v dT/T) + (\partial P/\partial T)_v dv \tag{3}$$

The equations of state[2] and functions[3] from Freon 14, which is the gas used in all the analyses, are

$$P = \frac{RT}{v} + \frac{(B_0RT - A_0 - C_0/T^2)}{v^2} + \frac{(bRT - a)}{v^3} + \frac{\alpha a}{v^6} + \frac{c(1 + \gamma/V^2)e^{-\gamma/V^2}}{v^3T^2} \tag{4}$$

$$C_v = a_4 + b_4T + c_4T^2 + d_4T^3 - J\left[\frac{d_3TK^2e^{KT}}{v - b}\right] - J\left[\frac{g_3TK^2e^{KT}}{2(v - b)^2}\right] \tag{5}$$

The references should be consulted for the applicable constants (B_0, A_0, C_0, b, a, α, c, γ, a_4, b_4, c_4, d_4, d_3, K, b, g_3). $[\partial P/\partial T]_{2=}$ and $[\partial P/\partial v]_T$ are obtained by differentiating the equation for P.

The foregoing thermodynamic equations are utilized by converting them to finite-difference forms by simply using ΔE for dE, etc. The approach used to determine the change in properties during a process is to add or subtract a small increment of mass ($\dot{m}$), thereby changing the specific volume; the work done by, or on the gas is calculated ($\Delta E_c = pv\dot{m}/J$), and Eq. (1) or (3) is then solved for ΔT. The gas is heated or cooled by accounting for the heat transfer at constant v.

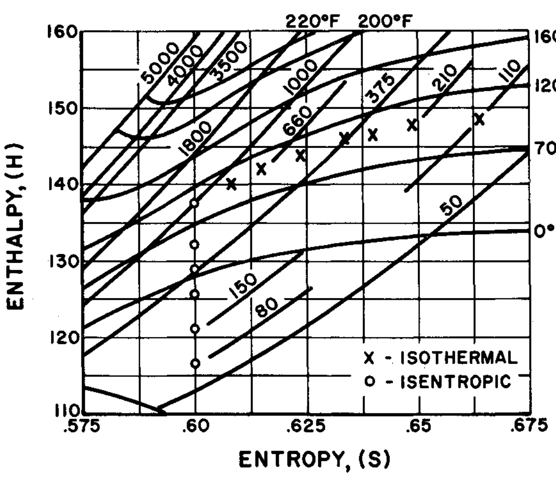

Fig. 2 "Freon 14" (*H-S* diagram).

For compression, the energy difference (ΔE_T) between incoming gas and tank gas must be calculated using Eq. (1); any desired accuracy can be achieved through selection of the temperature increment. This difference, combined with the work energy done in compressing the system, thus becomes the total energy ($\Delta E_T + \Delta E_C$) of the system ($\dot{m} + M$). The difference (energy) is now treated as the difference in energy between one state and the next state; the only unknown now in Eq. (1) is T, and it may be solved for directly to determine the new T:

$$T = \frac{\Delta E_t + \Delta E_c}{C_v(M + \dot{m})} - \left[T\left(\frac{\partial P}{\partial T}\right)_v - P\right]\frac{\Delta v}{C_v} \tag{6}$$

Iterations may be made on the step to utilize mean properties in all calculations of the step for accuracy. Further energy exchanges may be accounted for by heat transfer to a boundary, of which the temperature is known, or heat transfer to a container to maintain it at gas temperature. The heat transfer is accounted for simply by $dQ = C_v dT$ or $Q = C_v\Delta T$ since $\Delta v = 0$. All properties are calculated at the end of each step.

The analysis for expansion of the gas in a tank first calculates the change in v ($\dot{m}$ removed from total volume) and then uses Eq. (3) assuming constant entropy ($dS = 0$). The change in temperature is directly calculated to obtain the state for an isentropic expansion. This is done for all types of processes:

$$dT = -(T/C_v)(\partial P/\partial T)_v dv \tag{7}$$

If the expansion is to be other than isentropic, the appropriate energy is then added ($dQ = C_v dT$) to account for the required input or output for either an isothermal expansion or any arbitrary heat transfer. If the expansion is to be isentropic, the energy addition step is omitted. At the end of each step, the properties are all calculated (P, ΔH, H, ΔS, S, and Q from ΔV, V, ΔT, and T).

The impulse calculations are derived directly from the energy equation for fluid flow, from which ideal impulse can be shown to be

$$I_{\text{ideal}} = \dot{m}[2Jg\,(H_1 - H_2)]^{1/2} \tag{8}$$

and

$$I_{\text{spideal}} = W\,(2J/g\Delta H)^{1/2} \tag{9}$$

$$= 6.955\,(\Delta H)^{1/2} \text{ unit propellant weight)} \tag{10}$$

For a real nozzle (divergent with half-angle α), still flowing under ideal conditions,

$$I_{\text{sp}} = [(1 - \cos\alpha)/2]\,I_{\text{spideal}} + [(P_e - P_a)/W]\epsilon \tag{11}$$

Knowing the flow conditions, the flow and exit pressure can be calculated, and thus so can the exit enthalpy. If it is assumed that there is constant enthalpy across the flow control or pressure-regulating device, then the specific impulse of the nozzle can be determined as a function of enthalpy in the gas storage tank. The step can be carried further by obtaining nozzle coefficients and efficiencies experimentally.

Results and Conclusions

From Figs. 1 and 2 it is seen that the agreement between the predicted curves and the Freon 14 data is good. Figure 1 compares the calculated results with experimental data. The test data show the two temperature points that differ; this is due to stratification resulting from the cold inlet stream entering near the probe sensor and settling to the bottom; the prediction is seen to be basically in agreement with the test results, however. Test results are not available for the expansion; however, the accuracy of the prediction can be determined by plotting on the *H-S* diagram. An isentropic or isothermal expansion should plot as shown in Fig. 2, which indicates the accuracy of the prediction. Actually, the predictions are nearly exact; they are limited only by the accuracy of the equation of state, and the lack of perfect agreement is due more to experimental error than anything else. The pressure and temperature sensor are accurate to about $\pm 2\%$ of the range shown; the equation of state used is capable of predicting pressure within $\pm 0.5\%$ of its range (5000 psi), given temperature and specific volume.

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Correlation of Radar Attenuation with Rocket Exhaust Temperature

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Nomenclature

a_j	= exit plane radar attenuation
A, B, D	= terms defined by Eq. (6)
c^*	= characteristic velocity of propellant
C_r	= recombination rate parameter $\equiv \alpha_{r,c} n_{e,c} d_t/c^*$
d	= local diameter nozzle
k	= $d \ln(a_j/n_{e,j}d_j)/d\, \mathrm{in} \nu_{e,j}$; cf.,Eq. (6d)
M	= local Mach number
n	= effective pressure exponent in solid propellant (+ additive) burning rate law
n	= total number density
n_e	= electron number density
p	= local pressure; $\bar{p} \equiv p/p_c$
R	= universal gas constant
T	= local static temperature
V	= ionization potential of the ion's parent molecule
V_a	= apparent ionization potential; $V_a \equiv -2R\, d(\ln a_j)/d(1/T_j)$
X_e	= electron mole fraction n_e/n; $\bar{X}_e \equiv X_e/X_{e,c}$
α_r	= recombination coefficient
γ	= specific heat ratio (assumed constant)
θ_N	= half-angle of conical expansion section
ν_e	= electron-neutral collision frequency
ω	= (circular) frequency of electromagnetic radiation
ω_p	= plasma frequency

Subscripts

a	= apparent
C_r	= at constant C_r
c	= in chamber
e	= pertaining to electrons
eq	= in local thermochemical equilibrium
j	= at nozzle exit plane
V/RT_c	= at constant V/RT_c
∞	= ambient

Introduction

THE use of high-energy propellants and propellant additives can increase tracking and communication difficulties experienced when electromagnetic waves pass through a rocket exhaust plume. Attempts to understand the causes are inevitably complicated by the need to account for many simultaneous physicochemical and gasdynamic effects.[1–5] However, as a step in this direction, we consider those limiting cases amenable to theoretical analysis. In the present case, we shall assume that the dominant effect of a change in propellant formulation is to alter the combustion chamber temperature T_c and, hence, the extent of thermal ionization of an alkali-metal impurity already present in the propellant. Even here, a number of distinct phenomena must be included simultaneously, viz., the effects of altered T_c and p_c on 1) the equilibrium electron number density $n_{e,c}$ in the chamber, 2) the point within the nozzle at which the electron concentration "freezes," and 3) the collision frequency $\nu_{e,j}$ of electrons at the exit plane. Our limited object is to predict the local slope of a plot of the logarithm of exit-plane radar attenuation vs the reciprocal of the exhaust gas temperature for a fixed-geometry motor, and to define the conditions under which the resulting "apparent ionization potential" V_a bears a simple relationship to the actual ionization potential V of the alkali impurity atom.

Analysis

In traversing each unit length of rocket exhaust, an electromagnetic signal of prescribed frequency ω is attenuated by an amount depending on two plasma properties of this gas[7,8]: the plasma frequency ω_p ($\propto n_e^{1/2}$) and the electron-neutral collision frequency ν_e. Consequently, to predict V_a it is necessary to examine the nature of the attenuation law $a_j(\omega, \omega_p, \nu_e)$ and the way in which the exit-plane electron number density $n_{e,j}$ and total collision frequency $\nu_{e,j}$ respond to changes in T_c (and, hence, T_j). For this purpose, it will suffice to focus attention on functional form as opposed to absolute values. (Use will be made of the author's nonequilibrium nozzle flow theory,[3,4] to which the reader is directed for a more detailed exposition of the underlying assumptions.) Pressure dependencies are retained since, for solid propellant systems, a change in T_c will change p_c by an amount related to the effective pressure index n.[9] By equating the mass rate of hot gas generation ($\propto p_c^n$) to the mass flow rate through a supercritical nozzle ($\propto p_c T_c^{-1/2}$), one obtains the power law $p_c \propto T_c^{1/2(1-n)}$, which is incorporated in the following analysis and is taken to define n (which may be changed by an additive). By formally setting $n = -\infty$, wherever it appears, the results of the present analysis may be applied to (liquid-propellant) systems in which a change in p_c need not accompany a change in T_c. The effects of small changes in γ and mean molecular weight are neglected throughout.

The exit plane attenuation a_j is taken to be proportional to the electron "optical" depth, $n_{e,j}d_j$, in accord with the approximate law[4,7]

$$a_j \propto (n_{e,j}d_j) \cdot [\nu_{e,j}/(\omega^2 + \nu_{e,j}^2)] \tag{1}$$

Although $\nu_{e,j}$ is linearly proportional to p_j, its temperature dependence (at constant composition) is determined by that of the electron-neutral cross section for the dominant electron scatterer in the gas. This species is $H_2O(g)$ in the cases discussed below, so that [7] $\nu_e \propto p T^{-3/2}$. It is convenient to consider $n_{e,j}$ as the product of the total number density ($n_j \propto p_j T_j^{-1}$), the equilibrium electron mole fraction in the chamber, and $\bar{X}_{e,j} \equiv X_{e,j}/X_{e,c}$, which accounts for the electron-ion recombination lag during the nozzle expansion, i.e.,

$$n_{e,j} = n_j \cdot X_{e,c} \cdot \bar{X}_{e,j} \tag{2}$$

Subject to the assumptions of 1) weak ionization of the impurity, and 2) negligible negative ion concentrations, the Saha equation provides the functional dependence

$$X_{e,\mathrm{eq}} \propto p^{-1/2} T^{5/4} \exp(-\tfrac{1}{2}\, V/RT) \tag{3}$$

Local ionization equilibrium is approximately maintained in the nozzle until a region is reached beyond which the electron

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